Mathematics 2204 / 2205 Midterm Review

1. Solve:

a) $\begin{cases} y = -2x + 4 \\ 2x - 3y = 12 \end{cases}$

b) $\begin{cases} 2x - 5y = -10 \\ x + 2y = 13 \end{cases}$

c) $\begin{cases} 3x - 10y = 31 \\ 7x - 2y = 51 \end{cases}$

d) $\begin{cases} 8x - 6y = 0 \\ x + 9y = \frac{13}{4} \end{cases}$

e) $\begin{cases} \frac{2}{3}x + \frac{1}{4}y = 18 \\ \frac{1}{6}x - \frac{3}{8}y = -6 \end{cases}$

f) $\begin{cases} 2x - y = 13 \\ 3x + 4y = -8 \end{cases}$

2. Solve:

a) $\begin{cases} 4x + y + 2z = 14 \\ 2x - y - 2z = -6 \\ -x + 2y - z = -4 \end{cases}$

b) $\begin{cases} 5x + 3y - 4z = 1.9 \\ 2x - 3y - 8z = -69.1 \\ 6x + 2y - 7z = -22 \end{cases}$

c) $\begin{cases} 3x - 4y + 2z = -2 \\ 5x - y + 3z = 1 \\ x + y - z = -5 \end{cases}$

d) $\begin{cases} x + y + z = 20 \\ 0.1x + 0.15y + 0.25z = 4.4 \\ -4x + z = 0 \end{cases}$

3. One phone plan has $10 monthly fee and a 10 cent per minute charge and a second has a $20 monthly fee and an 8 cent per minute charge. For how many minutes would each plan charge the same amount?

4. Bob investigated three different phone plans. He learned that if he talked on the phone for 250 minutes a month, all three plans would cost the same, but they would not cost the same for other numbers of minutes. Write equations to describe the charges for each plan.

5. Write the following system as matrix multiplication. DO NOT SOLVE

$$\begin{aligned} 4a + 2b - 3c + d &= 1 \\ 4a - 3c &= 4 \\ 2a + c - 2d &= -8 \\ 2b - 3c &= 6 \end{aligned}$$

6. A grain dealer sold 5 bushels of wheat, 2 bushels of corn and 3 bushels of rye for $167.50. To another customer, he sold 2 bushels of wheat, 3 bushels of corn and 5 bushels of rye for $177.50 and to a third customer he sold 3 bushels of wheat and 2 bushels of rye for $80.00. Determine the price per bushel for each type of grain.

7. Jack buys 7 bars and 5 bags of chips for $3.75. Lynn buys 3 bars and 4 bags of chips for $2.35. What are the individual costs of a bar and a bag of chips?

8. Mile One Stadium and the Glacier are having two upcoming events: Worm Races and Cat Rolling. The information in the table below describes the ticket sales for each event to date.

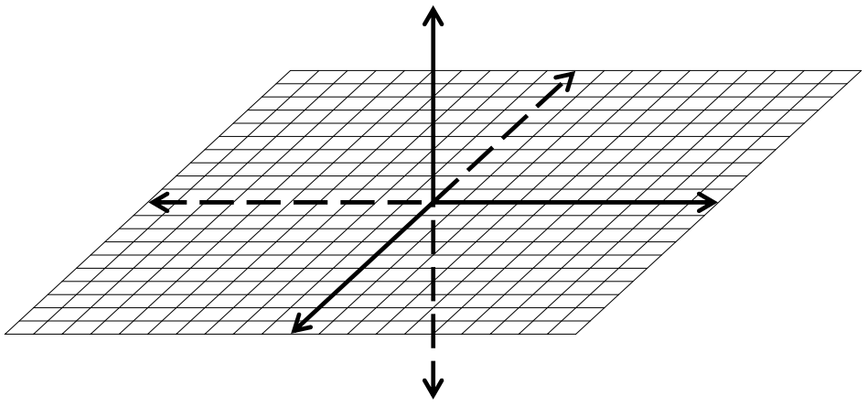
	# of tickets sold for Worm Races	# of tickets sold for Cat Rolling	Amount Collected ($)
Mile One	9	4	$47
Glacier	7	3	$36

a) Write equations that describe the situation.
b) Write the system using matrix multiplication.
c) Find the determinant of the coefficient matrix above and use it to determine the inverse of the coefficient matrix.
d) Solve the matrix equation to determine the number of tickets sold for each event. (DO NOT USE A GRAPHING CALCULATOR)

9. Graph the planes using the intercept method.

a) $-2x+y-3z=6$ b) $2x-3y+4z=24$ c) $3x+6y-9z=18$

10. Plot the following points on the grid below: A(2, -4, 1) B(-5, 3, -4) C(1, 6, 2)



11. Multiply the matrices below:

a) $\begin{pmatrix} 3 & 7 \\ 2 & 8 \end{pmatrix} \times \begin{pmatrix} 5 \\ 6 \end{pmatrix}$ b) $\begin{pmatrix} 2 & 5 & 3 \\ 4 & 4 & 2 \end{pmatrix} \times \begin{pmatrix} 1 & 3 \\ 2 & 2 \\ 3 & 1 \end{pmatrix}$ c) $\begin{pmatrix} 1 & 6 \\ 2 & 5 \\ 3 & 4 \\ 5 & 2 \end{pmatrix} \times \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

12. It has been shown that the amount of time a person exercises is related to the amount of water that he drinks by a quadratic function ($y = ax^2 + bx + c$). For one person those figures are given in the table:

Exercise Time	Water Consumed
1 hour	0.85 liters
2 hours	0.95 liters
3 hours	0.96 liters

Set up a 3 x 3 system that can be used to determine the quadratic function that is referred to above. (DO NOT SOLVE)

13. A baseball pitching machine shoots a baseball to the batter who is standing 27 m away. The ball is at a height of 1.0 m as it passes the batter. It leaves the machine at a height of 0.5 m and when it is 14 m away from the machine the ball is at a height of 6.24 m. Find a quadratic function which describes the path of the ball and use it to determine the height of the ball when it is 8 m from the machine.

14. Given the following information for a kiddie Ferris wheel at a local amusement park:

Time (s)	0	5	10	15	20	25	30
Height above ground (m)	6	10	6	2	6	10	6

a) Sketch a graph to represent the data in the table.
b) What is the height of the axel of the Ferris wheel?
c) What is the radius of the Ferris wheel?
d) How long does it take to make one complete revolution?
e) In the first 30 seconds, how many times is the height 8 m above the ground?
f) At what time in the first 30 seconds is the height 4 m above the ground?
g) Predict the height after 2 minutes.

15. Sketch each of the following using the mapping rule method:

a) $-\frac{1}{2}(y+3)=\sin 3(x-45°)$ b) $\frac{1}{4}(y-1)=\cos 2x$

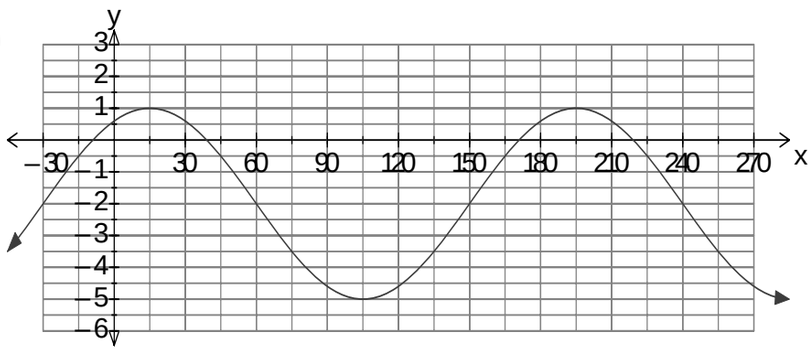
16. Sketch each of the following using the 5 point method:

a) $\frac{1}{3}(y-2)=\sin(x+60°)$ b) $-(y+4)=\cos 2(x+30°)$

17. For each function in #15 and #16 state the domain and range.

18. Write a function for each of the following graphs as a transformation of $y=\sin x$ **and** $y=\cos x$.

a)



$y=\sin x$ $y=\cos x$

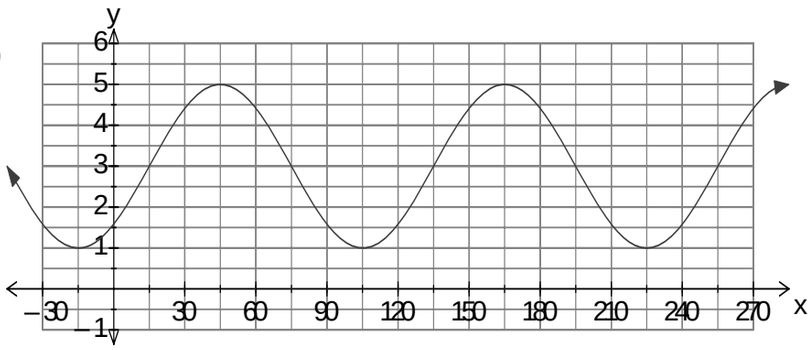
HS = HS =

HT = HT =

VS = VS =

VT = VT =

b)



$y=\sin x$ $y=\cos x$

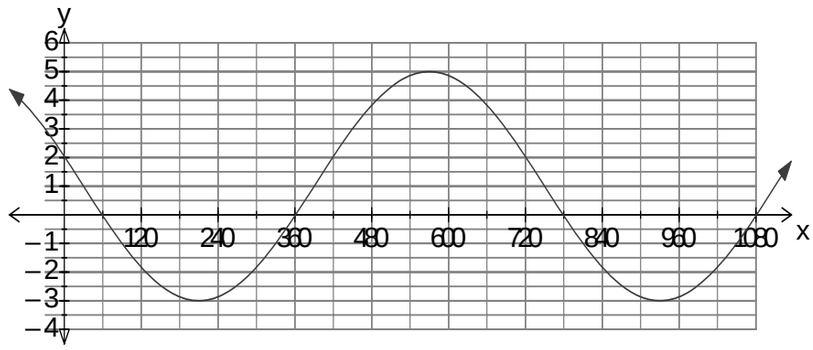
HS = HS =

HT = HT =

VS = VS =

VT = VT =

c)



$y=\sin x$ $y=\cos x$

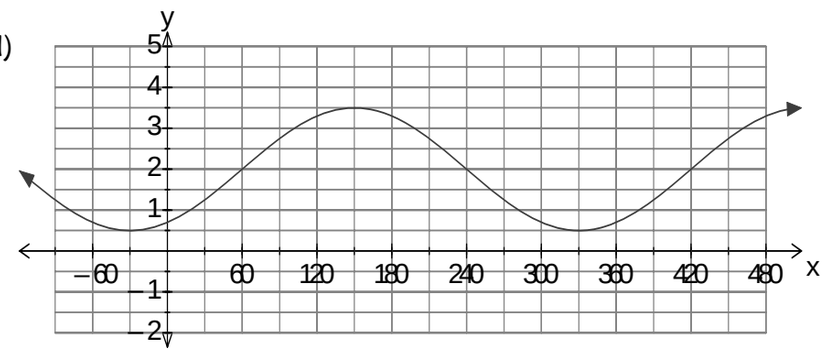
HS = HS =

HT = HT =

VS = VS =

VT = VT =

d)



$y=\sin x$ $y=\cos x$

HS = HS =

HT = HT =

VS = VS =

VT = VT =

19. Simplify. Express your answers in exact radical form.

a) $\dfrac{2\sin 60° + \cos 45°}{\sin(-135°)}$ b) $\dfrac{\sin 180° + \cos 390°}{\sin(-45°) - \sin 270°}$

c) $\dfrac{\cos 120°}{\cos 330°} - \dfrac{\sin 90°}{\cos 315°}$ d) $\dfrac{\sin^2 210°}{3\cos^2 120° + \cos 150°}$

20. Give a positive and negative co-terminal angle for each angle.

a) -142° b) 385° c) 220°

✻ 21. Solve each system.

a) $\begin{cases} y = 2x \\ y = x\sqrt{3} + 8 - \sqrt{48} \end{cases}$ b) $\begin{cases} y = 4x \\ y = -x\sqrt{32} + 2 - \sqrt{12} \end{cases}$

22. Name a point that lies on the plane $3x - 8y + 2z = 4$.

23. Which matrices do not have inverses? Explain why.

a) $\begin{bmatrix} 3 & -4 & 0 \end{bmatrix}$ b) $\begin{bmatrix} -1 & 2 & 4 \\ -2 & 0 & 9 \\ 2 & 6 & -4 \end{bmatrix}$ c) $\begin{bmatrix} 5 & 2 \\ 10 & 4 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

24. a) What is the local maximum of $y = -5\sin(x - 4) + 6$?

b) What is the local minimum of $y = -\frac{2}{3}\cos(x + 45) - 3$?

c) What is the equation of the sinusoidal axis of a graph having a local minimum of –9 and local maximum of 4?

d) What is the horizontal translation of $y = \cos x$ in the image graph of $4(y - 8) = \cos(5x - 30)$?

e) What is the period of $\frac{1}{3}(y + 9) = \sin(\frac{1}{6}x - 4)$?

25. What is the mapping rule for a rotation of a) 300° b) -270° ?

26. Determine the value of a) $\sin 38°45'$ b) $\cos 123°36'$.

27. Write a system of equations that is equivalent to $\begin{cases} 3x - 10y = 16 \\ -4x + y = -9 \end{cases}$.

28. Devise two telephone plans for which 120 minutes of long distance calls per month cost the same, but for other numbers of minutes the cost is different. Write equations for the telephone plans.

✻ 29. A triangle has vertices (4,2), (-2,-1), and (6,0). Determine the orthocentre of the triangle.

✻ 30. The tire of a bicycle has a radius of 26 cm. A nail becomes embedded in the tire as Jamie is completing his paper route. Determine the equation of the function that describes the height of the nail above the ground with respect to the distance the bicycle travels from the point where the nail became embedded in the tire. What is the height of the nail after the bicycle has covered a distance of 25 metres?

31. Factor each of the following:

(a) $x^2 - 100$ (b) $8x^2 + 12x^5$ (c) $x^2 + 9x + 14$ (d) $x^2 - 5x - 24$

(e) $x^2 + 6x - 16$ (f) $4y^2 - 36x^2$ (g) $3m^3 + 15m^2 - 18m$ (h) $2x^2 + 7x + 3$

(i) $5t^2 + 14t + 8$ (j) $4x^2 + 6x - 18$

32. Simplify the following rational expressions, and identify the *restrictions*.

(a) $\dfrac{x^2 + 12x + 20}{x^2 + 7x + 10}$ (b) $\dfrac{x^2 - 25}{x^2 + 3x - 40}$ (c) $\dfrac{4x^2 - 1}{2x^2 - 13x - 7}$

33. Simplify the following rational expressions, and identify the *restrictions*.

(a) $\dfrac{12x + 18}{x^3 + 4x^2} \times \dfrac{x + 4}{6x^2 + 9x}$ (b) $\dfrac{x^2 - 9}{x + 2} \div \dfrac{x^2 + 6x + 9}{2x + 4}$

(c) $\dfrac{3}{x - 1} + \dfrac{2}{x}$ (d) $\dfrac{2}{x + 2} - \dfrac{1}{x - 2}$

(e) $\dfrac{x^2 - 1}{2x^2 + 5x + 1} \times \dfrac{10x + 15}{x^2 + 4x - 5}$ (f) $\dfrac{2x}{x^2 - 9x + 18} + \dfrac{4x}{x^2 - 9}$

34. Use the unit circle to find the exact value for each of the following.

(a) sin 135° (b) cos -240° (c) tan 300° (d) csc -225° (e) sec 150° (f) cot -45°

(g) sec $-\pi$ (h) csc 3π (i) tan $\dfrac{-35\pi}{6}$ (j) cot $\dfrac{19\pi}{4}$ (k) $\sin^2\left(\dfrac{2\pi}{3}\right)$ (l) $\cos^2\left(\dfrac{-5\pi}{6}\right)$

35. Simplify each of the following.

(a) cos x · csc x · tan x (b) $\dfrac{\tan\theta + \cot\theta}{\cos\theta \sin\theta}$ (c) $\cos^3 x + \cos x \cdot \sin^2 x$

(d) $\dfrac{\cot^2\theta}{1 + \csc\theta} + \sin\theta \csc\theta$ (e) $\dfrac{\sec\theta + \csc\theta}{1 + \tan\theta}$

36. Prove each of the following identities.

(a) csc x (csc x + cot x) = $\dfrac{1}{1 - \cos x}$ (b) sin x tan x + cos x = sec x

(c) $\cos^2 x$ (1 + $\tan^2 x$) = 1 (d) $\sec^2 x + \csc^2 x = \sec^2 x \csc^2 x$

(e) (1 - tan x) (1 + cot x) = (cot x - 1)(tan x + 1)

(f) $\dfrac{\sin^2\theta}{\tan\theta + \sin\theta} = \dfrac{1 - \cos\theta}{\tan\theta}$ (g) $\dfrac{\sin\theta}{\sin\theta + \cos\theta} = \dfrac{\tan\theta}{1 + \tan\theta}$

(h) $\dfrac{\sin\theta\cos\theta}{1 + \cos\theta} - \dfrac{\sin\theta}{1 - \cos\theta} + \cot\theta\cos\theta + \csc\theta = 0$

37. Solve each of the following trigonometric equations for ALL possible values of x.

(a) sin x = 0.4 (b) cos x = -0.52 (c) cot x = -3 (d) sec x = 5

(e) cos x = $\dfrac{-1}{2}$ (f) sin x = $\dfrac{\sqrt{3}}{2}$ (g) $\sqrt{2}$ cos x = 1 (h) 2 sin x - $\sqrt{2}$ = 0

(i) cot x = $\sqrt{3}$ (j) tan x = 1 (k) tan x = cot x (l) sec x = cos x

(m) 3 cos x - 3 = 0 (n) -5 sin x - 5 = 0 (o) cos x = 3 (p) csc x = $\dfrac{1}{2}$

✤ 38. Solve each of the following trigonometric equations for $0 \le \theta < 2\pi$.

(a) $2\cos^2\theta + \sin\theta + 1 = 0$ (b) $2\cos^2\theta = \cos\theta$ (c) $\cos^2\theta - \cos\theta = \sin^2\theta$

(d) $\sin^2\theta - 5\sin\theta - 6 = 0$ (e) $2\cos^2\theta + \cos\theta - 1 = 0$ (f) $\cot\theta = 2\cos\theta$

(g) $\cos\theta\tan\theta = \cos\theta$ (h) $\tan^2\theta = 2\tan\theta\sin\theta$ (i) $2\csc^2\theta = 3\cot^2\theta - 1$

39. Solve each triangle ABC (i.e. find all missing angles and sides):

a) AB = 7, BC = 10, and $\angle B = 67°$ b) AB = 11, BC = 7, AC = 9.

40. From the top of cliff 40 m high the angle of depression to a town below is 44°. How far apart is the base of the cliff and the town?

41. Find the area of a parallelogram with sides 8 cm and 12 cm and a contained angle of 60°.

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